

7.2.1 Seismic waves

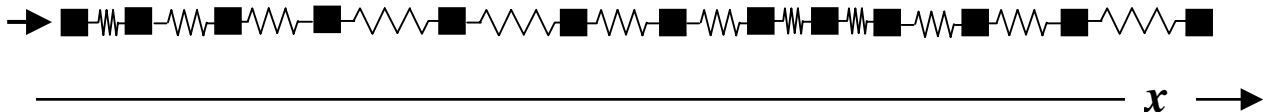
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Waves in a mass- spring system

A simple physical model that is useful for describing the motion of a sound wave in a gas or fluid is shown in the following cartoon. The density of the medium is represented by the distributed masses, and the compressibility is represented by the distributed springs. The masses and springs should be envisioned as lying in a frictionless ‘track’ which confines any motion to the x coordinate direction.

F



In this thought experiment the first mass is displaced to the right by an external impulsive force F . The mass accelerates to the right, compressing the spring and transmitting the force to the second mass. Meanwhile the external force is removed and the compressed first spring returns the first

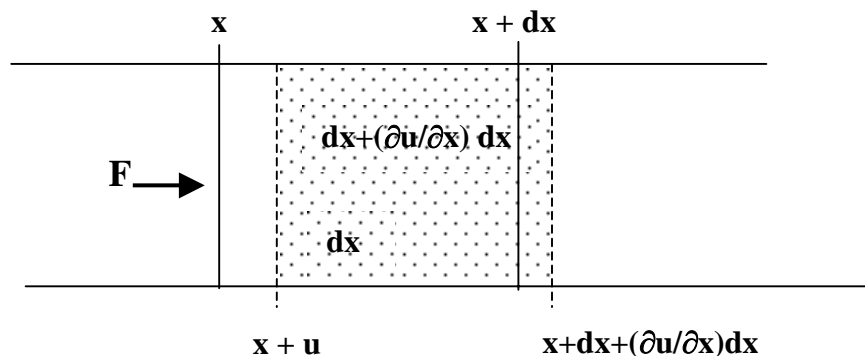
mass towards its initial position while acting on the second mass to transmit the forces to the right essentially initiating a continuous sequence of mass displacements along the chain of masses. Examined in time, any individual mass would be observed to undergo a short oscillatory movement in the x direction as the disturbance moved along the chain. A snapshot in time would show a chain of compressions and extensions of the springs as in the above cartoon. Successive snapshots in time of the whole chain would show the disturbance progressing from left to right as a wave. A particular state of, say, compression, called a point of constant phase of the wave, propagates to the right with the phase velocity, V_{ph} .



Acoustic waves in a liquid or gas

A quantitative analysis of this phenomenon in a liquid illustrates the relationship between the wave and the density and compressibility of the liquid or gas.

Consider two planes at x and $x + dx$ perpendicular to the x axis with a uniform force applied in the x direction on the plane at x . A small patch on these planes has area A . The volume $A dx$ is an *element* of the medium.



The force applied to the plane at x causes its displacement to $x + u$ which in turn causes the second plane to be displaced by a different amount given by the rate of change of the displacement with x , times the separation of the planes or $(\partial u / \partial x) dx$. The strain, the change in dimension in the x direction normalized by the unstressed dimension in x , is then simply $\partial u / \partial x$.

The stress at x is P , which is the force per unit area or F/A . The force also varies with x so we may write the stress at $x + dx$ as $P + (\partial P / \partial x) dx$. Hooke's law relates the stress and strain through the elastic constant of the medium, in this case through the bulk modulus K . So writing the stress at x in terms of K and the above strain we find that:

$$P = K \times \text{strain} = K (\partial u / \partial x)$$

P changes with x and so at $x = x + dx$,

$$\begin{aligned} P &= P + (\partial P / \partial x) dx \\ &= P + \partial / \partial x (K (\partial u / \partial x)) dx \\ &= P + K (\partial^2 u / \partial x^2) dx \end{aligned}$$

So there can be an unbalanced force, $A K (\partial^2 u / \partial x^2) dx$, acting on the element dx which causes it to accelerate. By Newton's Law this force must equal the mass of the element times its resulting acceleration, so

$$A K (\partial^2 u / \partial x^2) dx = \rho A dx \partial^2 u / \partial t^2$$

where ρ is the density of the medium.

So, finally, we have an equation of motion for the element,

$$\partial^2 u / \partial x^2 - \rho / K \partial^2 u / \partial t^2 = 0$$

Since we normally deal with oscillatory or quasi-harmonic disturbances we can explore the behavior of solutions to this equation by assuming that the solution in time is sinusoidal and that the time dependence is given by $e^{i\omega t}$,

where ω is the angular frequency in radians/sec (equal to $2\pi f$, where f is the frequency in cycles per second or Hertz). With this solution in t the equation of motion becomes:

$$\partial^2 u / \partial x^2 + \omega^2 \rho / K u = 0$$

or, with $\omega^2 \rho / K = k^2$

$$\partial^2 u / \partial x^2 + k^2 u = 0$$

The solution for the displacement u is,

$$u(x,t) = u_0 e^{i\omega t} e^{\pm i k x} = u_0 e^{i(\omega t \pm k x)}$$

This solution is basically the definition of a propagating wave. At a fixed position in x , a particular element of the medium oscillates back and forth in the x direction about a rest position (corresponding to the in-line oscillation of a particular mass in the above mass-spring analog). At a fixed time the displacements are distributed sinusoidally along x just as the spring distortions were distributed in the mass-spring analog.

Another way to visualize the solution is to imagine the movement of a molecule or particle in the liquid. The particle movement would be back and forth along the x -axis, an oscillatory movement in the direction of propagation. At this microscopic level the masses in the mass-spring analog are like particles in the liquid.

The derivation for compressive-extensional waves in a thin rod is identical except that the bulk modulus is replaced by [Young's modulus, E](#).

The exponent $\omega t \pm k z$ is called the phase and we can easily see what is required to stay at a point in x and t where the phase is constant. When $\omega t \pm k z = C$, $\partial z / \partial t = \pm \omega / k$ which is the velocity of the phase point, the phase velocity V_{ph} . The plus sign is for a wave traveling in the plus x direction, the minus sign for one traveling in the opposite direction.

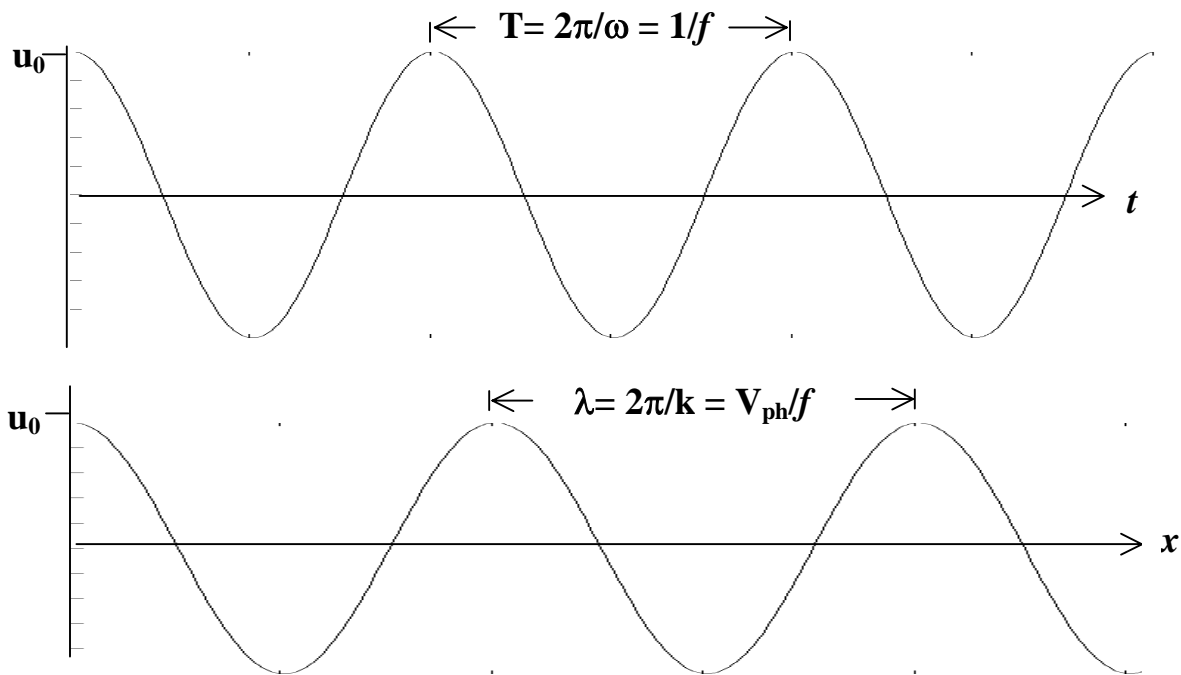
Substituting for k , the phase velocity is given by:

$$V_{ph} = (K/\rho)^{1/2}$$

The phase velocity is equal to the square root of the ratio of the bulk modulus to the density. Denser fluids thus have lower velocity than lighter fluids. Gases follow the same formula. The table below lists some densities and compressibilities and the derived velocities for some common liquids and air.

The distance at a given time between two successive points of equal phase on the wave is the wavelength, λ , and the time between two successive points of equal displacement at a given position on the x axis is the period, T , of the wave.

The velocity, frequency, period and wavelength are all related as can be seen in the following figure.





Seismic waves in a solid

The derivation of the equation of motion for an element in a solid is basically the same as that outlined above but it is much more complicated to the fact that the restoring elastic forces include shear as well as compression. The detailed analysis, done very well for example in Telford et al., shows that there exists a compressional wave with particle motion in the direction of wave propagation and also a shear wave with particle motion in the plane perpendicular to the direction of propagation. The velocities of these waves are given in terms of the [bulk and shear moduli](#), K , μ and density, ρ , viz.:

$$V_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \quad \text{and} \quad V_S = \sqrt{\frac{\mu}{\rho}}$$

It is clear from these expressions that the P-wave velocity is always larger than the S-wave velocity and that the S-wave is not viable in a medium with no shear strength.

Why the shear modulus is involved in the P-wave velocity is not intuitively obvious. The above derivation for the wave motion in a fluid is based on the fact that the bulk modulus K relates the volumetric strain to a *hydrostatic* pressure field. Although not depicted in the cross section the volume element is deformed equally in each coordinate direction – the moving pressure field does not change the *shape* of the element. In a solid the pressure disturbance is not hydrostatic and the shape of the element changes under deformation. It is the difference in deformation in different directions that brings the shear modulus into play in the resulting equations of motion.

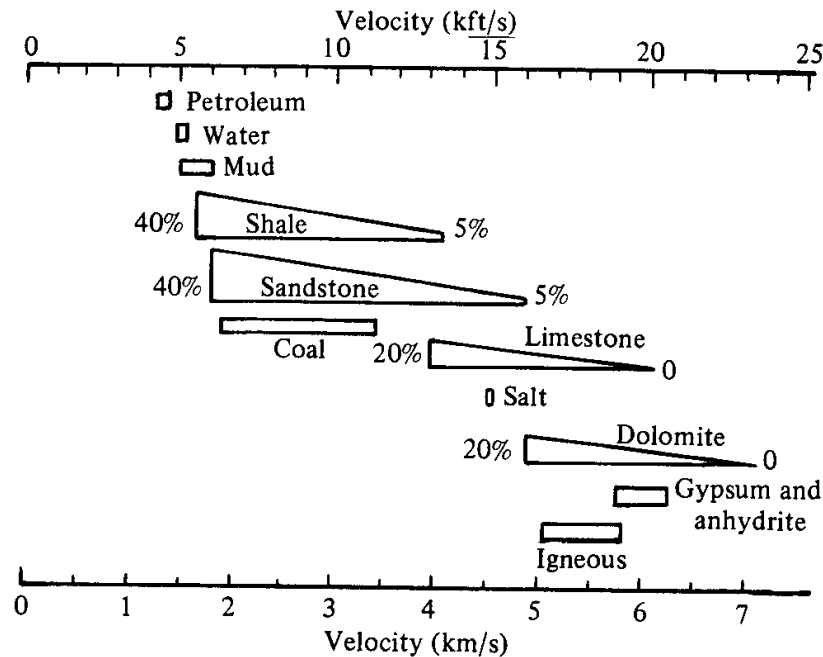
The particle velocity in a shear wave can be in any direction in the plane perpendicular to the propagation direction and it has become conventional to describe a shear wave by a component whose particle movement is in the horizontal direction, the S_H wave and a component whose particle motion is in a vertical plane, the S_V wave. Any S-wave propagating in any direction can always be represented by a superposition of S_H and S_V waves.

The [elastic constants](#) are all interrelated and a consequence that is very useful in the study of soils and rocks is that Poisson's ratio (ν) can be determined from the P and S wave velocities via the following equation;

$$\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad \text{and so,} \quad \nu = \frac{V_P^2 - 2V_S^2}{2(V_P^2 - V_S^2)}$$

Poisson's ratio is theoretically bounded between 0 and 0.5 and for most rocks lies around 0.25, so typically V_P/V_S is about 1.7.

The values of P wave velocities in representative crustal rocks are shown in the following Figure:

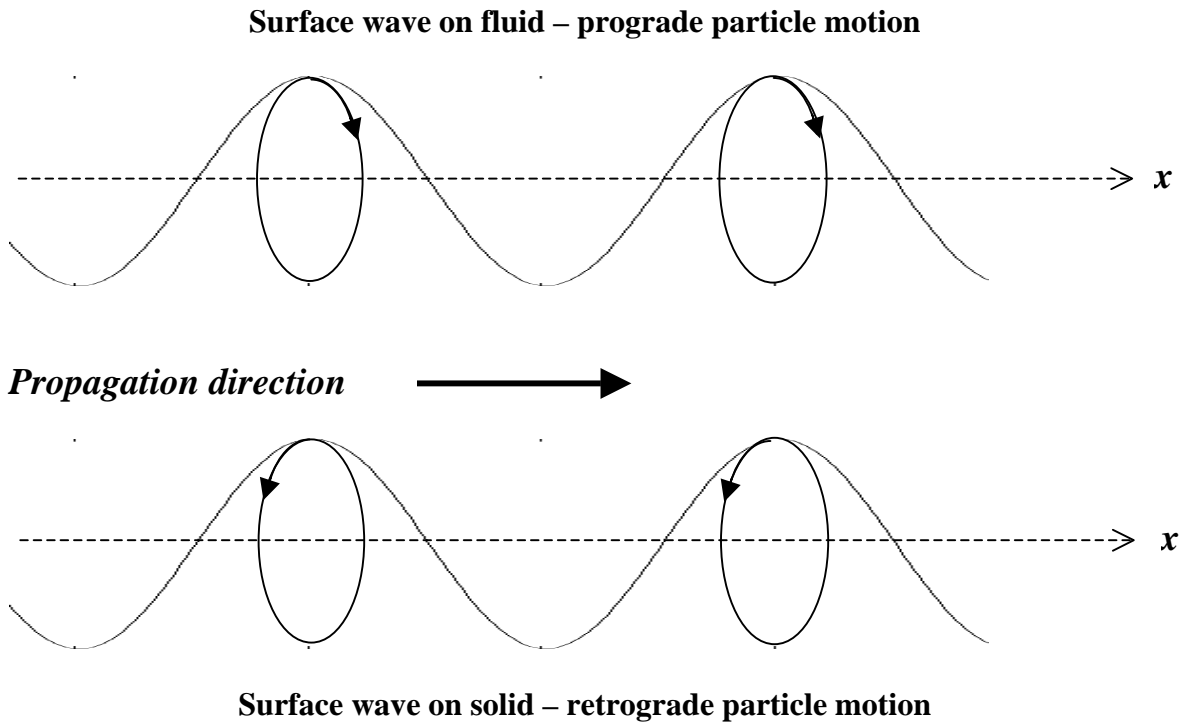


Surface waves

On the surface between a solid or liquid and air the deformation of an element is unconstrained in the direction normal to the surface. This changes the equation of motion for elements or particles adjacent to the surface and gives rise to another wave type commonly called the surface wave. Water waves are familiar and are readily seen as having a much lower velocity than a sound wave propagating through the bulk of the medium. Further a particle moves up and down vertically as well as in the direction of the wave. In detail a particle traces an ellipse with a *prograde* rotation as shown in the sketch below.

The surface wave on an isotropic half-space is known as a Rayleigh wave and it is similar in form to the surface wave on a liquid half-space except that the particle motion is *retrograde*. Rayleigh waves carry a large

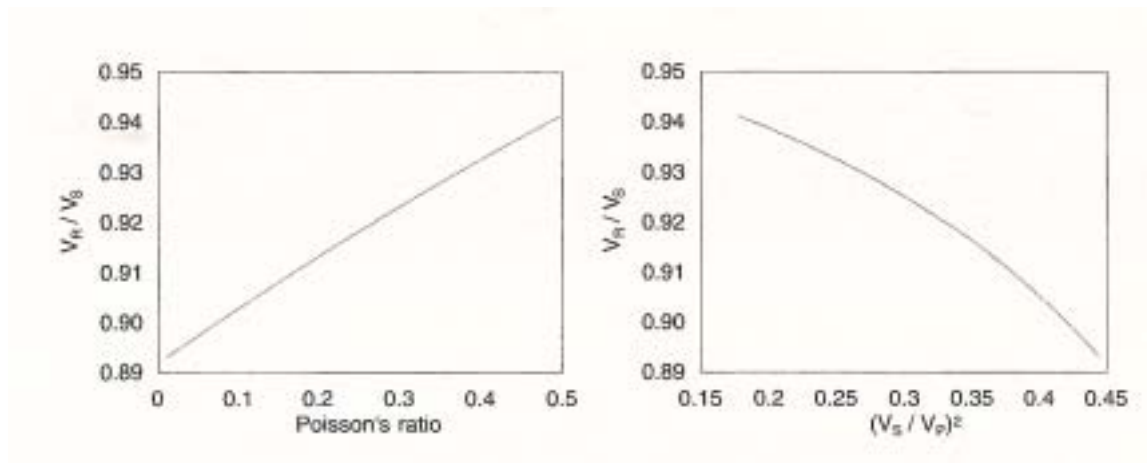
amount of energy away from surface sources and are the noticeable, or ‘felt’, ground motion when one is standing near a seismic source. In exploration seismology Rayleigh wave are known as ground roll.



The velocity of a Rayleigh wave, V_R , is tied to the S-wave velocity and Poisson’s ratio (and hence to V_P) through the solution to the following equation (White, 1983)

$$\left(2 - \left(\frac{V_R}{V_S}\right)^2\right)^2 - 4\left(1 - \left(\frac{V_R}{V_P}\right)^2\right)^{\frac{1}{2}}\left(1 - \left(\frac{V_R}{V_S}\right)^2\right)^{\frac{1}{2}} = 0$$

The two plots below are taken from Mavko et al., 1998.



They show that for typical values of Poisson's ratio the Rayleigh wave velocity varies only from 0.91 to 0.93 V_S .

The amplitude of the particle displacement for surface waves decreases exponentially beneath the surface and the exponent is proportional to the frequency as well as the elastic constants and density of the medium. Since it is observed that velocity generally increases with depth, Rayleigh waves of high frequency penetrate to shallow depth and have *low* velocity whereas Rayleigh waves of low frequency penetrate to greater depth and have *high* velocity. The change in velocity with frequency is known as *dispersion*. For a typical surface source of Rayleigh waves the initiating disturbance has a broad frequency spectrum so the observed surface motion at some distance from the source is spread out in time, with low frequencies coming first and high frequencies coming later.

Measured Rayleigh wave dispersion can be used to invert for the vertical velocity profile.

Finally, where the shear wave velocity increases with depth from the surface, Love showed that a horizontally polarized surface wave could exist. This wave, now called a Love wave, is also dispersive, and measurements of

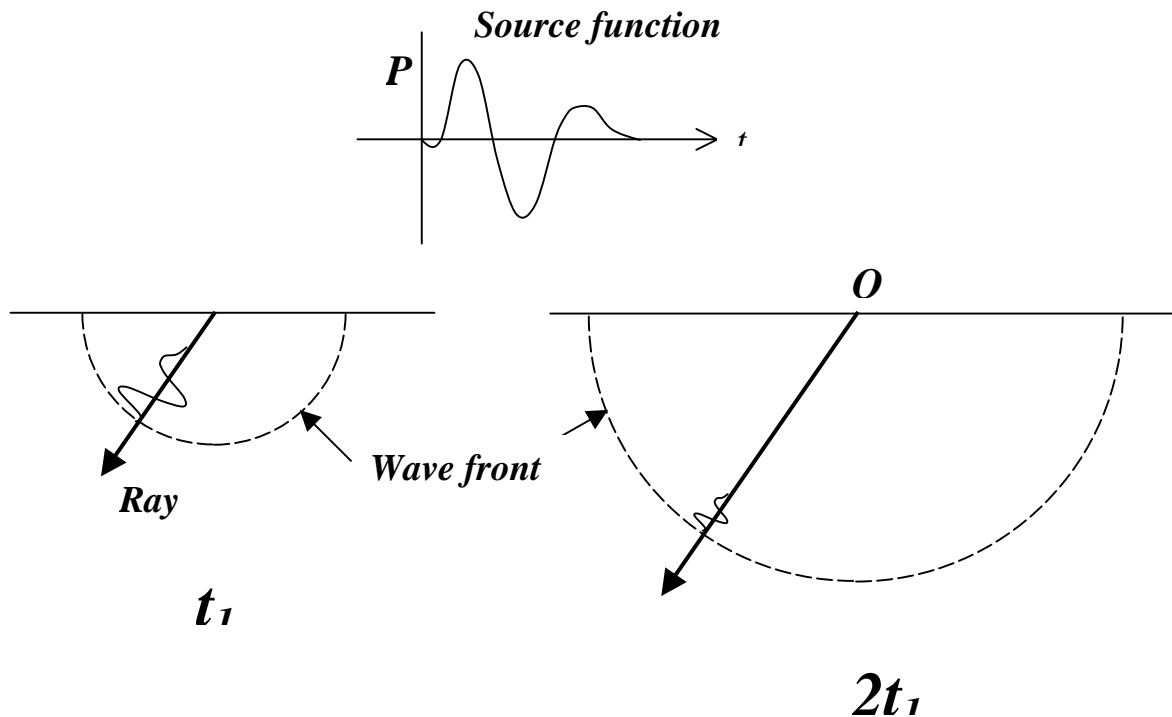
it's dispersion can also be inverted to find the distribution of shear wave velocity with depth. Love waves require a distinct shear component in the source stress field so they are not a strong component of the disturbance propagated from the explosive or surface impact sources used in seismic exploration. They can be, and are, generated by special horizontal shear sources. For many shallow earthquakes they may be the dominant surface wave. This is a major factor used in discriminating between underground nuclear explosions and shallow earthquakes.



Wavefronts, rays and geometrical attenuation

Until now we have concentrated on the physics of stress-strain fields and their propagation in homogeneous solids and liquids. The fields arising from actual natural or artificial sources and what happens to such waves when they encounter inhomogeneities in density or elastic moduli is the real subject of interest. These are the properties that we wish to extract from measurements of the waves that have propagated into the earth.

To begin, an assumption is made that a point source is responsible for the resulting seismic wave. In the case of an explosive source it is assumed that an impulse of radially directed compressive force is created at a point. Similarly a hammer hitting a small steel plate on the surface is considered a point source of vertical compressive force. Such simple concepts immediately run into practical limitations (what if the ground surrounding the explosive is itself inhomogeneous, or the charge is close to the free surface? The stress field of the steel plate is not hemispherical and there is, in practice a strong component of shear stress along the edges of the plate.).

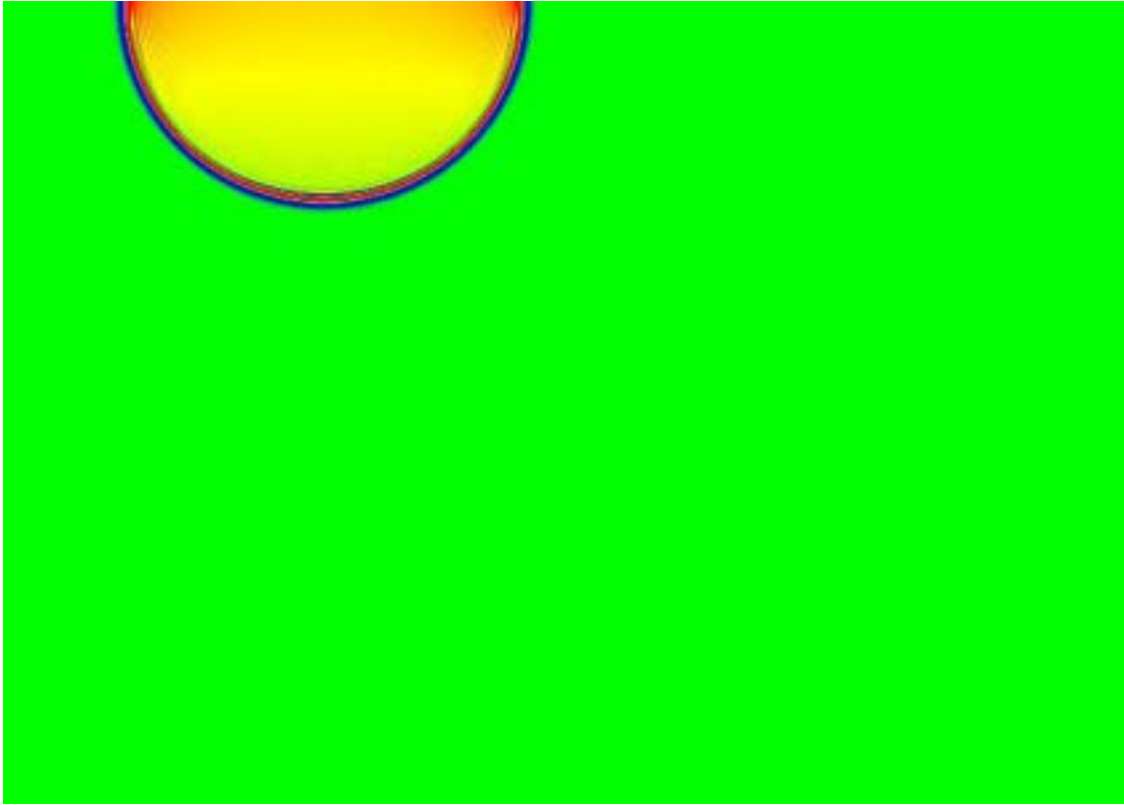


Considering the ideal compressive source at point O on a homogeneous half-space, the resulting seismic P-wave propagates such that all the points of constant phase lie on a hemisphere centered at O . The surface of all points of the same phase is called a wave front although the inferred meaning is that the phase in question is associated with some identifiable first arrival of the wave. A more rigorous definition is that the wave front is the surface of all equal travel times from the source. A cross section of such a hemispherical spreading wave is shown in the following cartoon for two successive time steps, t_1 and $2t_1$.

The wave in this cartoon is really a *wavelet*, the impulsive short wave train generated by the first compression of the source region, its subsequent recovery and perhaps a few highly damped oscillations. (The source deformation is large, not infinitesimal, and non-linear. Energy is dissipated in irreversible deformation (damage) to the surroundings, and the pulse dies quickly as suggested by the cartoon 'source function').

For convenience in describing wave propagation, the vector perpendicular to a wave front is defined as a *ray*. The ray in the cartoon is directed along a radius from the source, but this is not always the case. Rays are useful for describing what happens to waves when they pass through an interface and we will use them later to describe reflecting and refracting waves in layered media.

A numerical simulation of a seismic P-wave propagating away from a point source is shown in the following movie. In the movie the amplitude of the wavelet is contoured and color-coded. The high amplitude values are in red and the low amplitude in blue in this plot. The zero of the wavelet is the color of the undistorted background half-space. The time traces of geophones located in a well in the right hand side of the section are shown in the graph on the right of the figure. This is the fundamental form of the traces recorded in the vertical seismic profiling (VSP) method that we will return to later in the course.



[*Click here to start the movie*](#)



Amplitude and energy

The energy imparted to the wave is proportional to the square of the amplitude of the resulting propagated wavelet. Barring actual non-elastic losses in the medium, the energy is conserved but is spread over the hemispherical shell that contains the wavelet. The energy in a volume element of this shell at radius r is roughly the total energy of the disturbance, E_0 , divided by the area of the hemisphere, $2/3\pi r^2$. The energy in the wavelet thus falls off, or attenuates, as $1/r^2$. The energy in the wavelet is proportional to the square of the amplitude of the wavelet, so the amplitude of the spherically spreading wave falls off as $1/r$. The second wave front at

$2t_1$ in the above cartoon is twice as far from the source as the first and so the amplitude of the wavelet is reduced by half. This is called the geometric attenuation and it is separate from any other attenuation due to loss mechanisms in the medium.

Surface waves spread in circular patterns from a point source (think water waves spreading from the impact point of a stone) and in this case the energy decays as $1/r$ from the source and the amplitude consequently falls as $1/\sqrt{r}$.

At very large distances from the source, the curvature of the wave front becomes small and in some situations it is possible to neglect the curvature and assume that the wave front is a plane. Such a wave is then known as a *plane wave*.